BHARATIYA VIDYA BHAVAN'S V M PUBLIC SCHOOL, VADODARA

QUESTION BANK

CHAPTER -5. CONTINUITY AND DIFFERENTIABILITY

Answer the following.

1. If f(1) = 4 and f'(1) = 2, find the value of derivative of log f(e^x) w.r.t. x at the point x=0 .
2. If y = e^{ax} cos bx , prove that y_2 - 2 ay_1 + (a^2 + b^2) y = 0.
3. Write the derivative of sinx w.r.t. cosx.
4. If y = e^{-ax} cos (bx + c), show that y_2 + 2 ay_1 + (a^2 + b^2) y = 0.
5. If x = tan ($\frac{\log y}{a}$) ,prove (1 + x^2) $\frac{d^2y}{dx^2}$ + (2x – a)$\frac{dy}{dx}$ = 0 .
6. Find $\frac{d^2y}{dx^2}$ at θ = $\frac{\pi}{2}$, if x = a(1+ cosθ) and y = a(θ + cosθ).
7. If y= $(\tan^{-1} x)^2$, show that $(x^2+1)^2$ y"+ 2x (x^2+1) y' = 2.
8. If y= log $\left[x+\sqrt{a^2+x^2}\right]$, show that (x^2 + a^2) $\frac{d^2y}{dx^2}$ + x $\frac{dy}{dx}$ = 0.

9. Find the value of a, b, c so that the function f(x) defined below is continuous at x = 0

   a. f(x) = $\begin{cases} \frac{\sin(a+1)x+\sin x}{x}, & x>0 \\ c, & x=0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{\frac{3}{2}}}, & x<0 \end{cases}$

10. Differentiate the following with respect to x : $\sin^{-1}\left(\frac{3^x 2^{x+1}}{1+36^x}\right)$.
11. If $y = (\log x)^{\cos x} + \frac{x^2+1}{x^2-1}$, $find\ \frac{dy.}{dx}$
12. If = $tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$; show that $\frac{dy}{dx} = -\frac{x}{\sqrt{1-x^4}}$.

13. Find the value of k, for which the following function is continuous at x = 0.

f(x) = $\begin{cases} \frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & if\ -1 \le x < 0 \\ \frac{2x+1}{x-1}, & if\quad 0 \le x < 1 \end{cases}$

14. For what value of k the function , $f(x)=\begin{cases}\dfrac{\sqrt{5x+2}-\sqrt{4x+4}}{x-2} & ,if\ \ x\neq 2\\ k & ,if\ \ x=2\end{cases}$,

is continuous at x = 2 ? .

15. Find k, if the function f(x) = $\begin{cases}\frac{kcosx}{\pi-2x}\ ,\ when\ x\neq\frac{\pi}{2}\\ \frac{\sin 3x}{x}\ ,when\ x=\frac{\pi}{2}\end{cases}$ is continuous at x = π /2.

16. Find $\frac{dy}{dx}$, if x = 3 cos θ – 2 cos $^3\theta$, y = 3 sin θ – 2 sin $^3\theta$.

17. If y = tan $^{-1}$ ($\frac{a}{x}$) + log $\sqrt{\frac{x-a}{x+a}}$, prove that $\frac{dy}{dx}$ = $\frac{2a^3}{x^4-a^4}$.

18. Prove that the function f(x)= | x – 1 | , x $\in$ R , is not differentiable at x = 1.

19. The function f(x) is defined as f(x) = $\begin{cases}x^2+ax+b\ ,\ 0\leq x<2\\ 3x+2\ ,\ \ \ \ 2\leq x\leq 4\\ 2ax+5b\ ,\ \ 4<x\leq 8\end{cases}$ If f(x) is continuous on [0,8], find the value of a and b.

20. Find the value of k , if the function $f(x)=\begin{cases}\frac{2^{x+2}-16}{4^x-16}\ ,\ if\ x\neq 2\\ K\ \ \ \ ,\ if\ x=2\end{cases}$ is continuous at x=2.

21. Find $\frac{dy}{dx}$, if xy = $e^{(x-y)}$.

22. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to x.

23. If y = 3 cos(log x) + 4 sin(log x) , show that $x^2\ \frac{d^2y}{dx^2}$ + x $\frac{dy}{dx}$ = – y.

24. Find the derivative of the following:

(I) $(cosx)^x$ + $(sinx)^{\frac{1}{x}}$ (II) ($x)^{sinx}$ + ($sinx)^x$ (III) $(xcosx)^x$ + $(xsinx)^{\frac{1}{x}}$.

25. Verify Rolle's theorem for the function f(x) = $(x-2)^2(x-1)$ on [1, 2].

26. Differentiate y = $\sin^{-1}\left(\frac{2^{x+1}.\ 3^x}{1+36^x}\right)$ with respect to x .

27. Differentiate $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}x^2$.

28. Show that the function f(x) = $|x-3|$, xϵR is continuous but not differentiable at x=3.

29. If y =cosec $^{-1}$x , x> 1 , then prove that x $(x^2-1)\ \frac{d^2y}{dx^2}+(2x^2-1)\ \frac{dy}{dx}$ = 0.

30. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ with respect to $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$,when x≠0 and $\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$.

31. If f(x) = $\begin{cases} \frac{cos^2x - sin^2x - 1}{\sqrt{x^2+1}-1}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, find the value of k .

32. Discuss the differentiability of f(x) = |x-1| + |x-2|.

33. If x = cost + log tan $\frac{t}{2}$ and y = sint, then find the value of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at t = $\frac{\pi}{4}$.

34. Show that the function f(x) defined by f(x) = $\begin{cases} \frac{sinx}{x} + cosx, & x > 0 \\ 2, & x = 0 \\ \frac{4\left(1-\sqrt{1-x}\right)}{x}, & x < 0 \end{cases}$ is continuous at x=0.

35. Show that the function f(x) = $|x-3|$, xϵR is continuous but not differentiable at x=3.

36. If a $\tan^{-1} x$ = $\log y$, then prove that $(1+x^2)$ y" +$(2x-a)y'$=0

37.If x = a$cos^3\theta$, y = a$sin^3\theta$, find y" at $\theta = \frac{\pi}{2}$.

38. If y = $(\sin^{-1} x)^2$, Prove that (1-x^2)y" – xy'=2

39. Find the second order derivative of e^{6x}cos3x.

40. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

41.Verify Rolles theorem for the function f(x) = x^2 + 2 x - 8, x∈ [– 4, 2]

42. Verify Lagrange's Mean value theorem for the function f(x) = x +$\frac{1}{x}$ in the interval [1,3].

43. If $y\sqrt{x^2+1} = \log\left(\sqrt{x^2+1} - x\right)$, show that $(x^2+1)\frac{dy}{dx}$ + xy+1 = 0.

44. Discuss the continuity of the function $f(x) = \begin{cases} \frac{x^4+2x^3+x^2}{\tan^{-1} x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ at x = 0.

45.Find the values of a and b, if the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$

is differentiable at x =1

.

48. Verify Lagrange's mean value theorem for f(x) = (x-1)(x-2)(x-3) in [1,4].

49. Given that $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, & x<0\\ a, & x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x>0\end{cases}$. if f(x) is continuous at x = 0 , find a .

50. If $y=\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, *prove* $(1-x^2)\ y''-3x\ y'-y=0$

****