BHARATIYA VIDYA BHAVAN'S V.M.PUBLIC SCHOOL, VADODARA

STD: XII **SAMPLE PAPER: 3** **MAX MARKS:100**
MATHEMATICS **TIME : 3 HRS**

General Instructions:

- All questions are compulsory
- The question paper consists of 29 questions divided into three sections A, B , C and D.
- Section A contains 4 questions of 1 mark each.
- Section B contains 8 questions of 2 marks each.
- Section C contains 11 questions of 4 marks each.
- Section D contains 6 questions of 6 marks each.

SECTION A

1. Find the direction cosines of the line passing through origin and lying in the first octant, making equal angles with the three coordinate axes.
2. Evaluate the value of $\hat{i}\bullet(\hat{j}\times\hat{k})+\hat{j}\bullet(\hat{k}\times\hat{i})+\hat{k}\bullet(\hat{i}\times\hat{j})$,
3. Find the sum of order and degree of the following differential equation $5x\frac{dy}{dx}-\left(\frac{d^2y}{dx^2}\right)^3=-6y$.
4. Using determinants, find the area of the triangle whose vertices are P(1, 1), Q(2, 7) and R(10, 8).

SECTION B

5. Write the integrating factor of $\frac{dx}{dy}$ +(tan y) x = sec ^{2} y .
6. Write the distance of the plane 2x - y + 2z +1=0from the origin.
7. Evaluate : tan { 2 tan $^{-1}$ $\left(\frac{1}{5}\right)$ + $\frac{\pi}{4}$ } .
8. Find a matrix X ,such that : X $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$.
9. For the following matrices $A=\begin{bmatrix}1\\-4\\3\end{bmatrix}, B=\begin{bmatrix}-1 & 2 & 1\end{bmatrix}$,verify (AB) ′ = B ′A′ .
10. Find the inverse of the function f: N→N defined by f(x)= x ^{2} + x + 1, x ∈ N, if it exists
11. Find the inverse of $\begin{bmatrix}1 & 2 & -2\\-1 & 3 & 0\\0 & -2 & 1\end{bmatrix}$ using elementary transformations.
12. Show that the derivative of $tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ w.r.t. tan $^{-1}$ x is independent of x .

SECTION C

13. Find the value of k ,for which the function

$$f(x)=\begin{cases}\frac{k\cos x}{\pi-2x}, & x<\frac{\pi}{2}\\ 3 & ,x=\frac{\pi}{2}\\ \frac{3\tan 2x}{2x-\pi}, & x>\frac{\pi}{2}\end{cases}$$

is continuous at x = $\frac{\pi}{2}$.

14. If (ax + b) $e^{y/x}$ = x, then show that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$.

15. Evaluate $\int \frac{1}{\sqrt{5-4x-2x^2}}dx$

16. $\int \frac{1-x^2}{x\ (1-2x)}dx$.

17. Using properties of determinants , prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$.

Or

Using properties of determinants ,prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

18. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$,find a vector c ,such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \bullet \vec{c} = 3$.

19. Find the value of λ ,so that the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = y - 5 = \frac{6-z}{5}$ are perpendicular to each other.

Or

Find the equation of the plane passing through the line of intersection of the planes 2x + y – z = 3 , 5x – 3 y +4z +9=0 and is parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

20. Three cards are drawn from a pack of 52 cards. Find the probability distribution of the number of aces. Also find the mean of the distribution.

Or

Probabilities of solving a specific problem independently by A,B and C are ½ , 1/ 3 and 1/4 respectively. If they try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

21. Evaluate : $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}dx$.

22. Evaluate : $\int_1^4 [\ |x-1| + |x-2| + |x-4|\ ]\ dx$.

23. Find the image of the point with position vector $\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ $in\ the\ plane$ $\vec{r}.\left(2\hat{\imath} - \hat{\jmath} + \hat{k}\right) + 3 = 0.$

SECTION D

24. Using the method of integration , find the area enclosed by the lines 2x + y = 4 ; 3x – 2 y = 6 and x – 3 y +5 =0 .

Or

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$.

25. Solve the following differential equation: $(x^2 - y^2)$ dx + 2xy dy =0 , given y = 1 when x = 1.

Or

Solve the following differential equation : x log x $\frac{dy}{dx}$ + y = 2 log x .

26. Consider f: $R_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ .Show that f is invertible. Find the inverse of f .

Or

Let A = Q x Q .Let * be a binary operation on A defined by (a , b) * (c , d) = (ac , ad + b). Check if * is assosciative . Find identity element and invertible elements of (A,*).

27. Show that the isosceles triangle of maximum area that can be inscribed in a given circle is an equilateral triangle.

28. A diet for a sick person must contain atleast 4,000 units of vitamin ,50 units of minerals and 1,400 units of calories. Two foods X and Y are available at a cost of Rs. 4 and Rs 3 per unit respectively. One unit of food X contains 200 units of vitamins , 1 unit of minerals and 40 units of calories, whereas one unit of food y contains 100 units of vitamins , 2 units of minerals and 40 units of calories.Find what combination of X and Y should be used to have least cost satisfying the requirements

29. Of the students in a college ,it is known that 60 % reside in hostel and 40% day scholars.Previous year results report that 30 % of students who reside in hostel attain A grade and 20 % of dayscholars attain A grade in their annual examination.At the end of the year ,one student was selected at random and he has an A grade ,what is the probability that the student is a hosteler?

Prepared by Ms.S.Devasena