BHARATIYA VIDYA BHAVAN'S V.M.PUBLIC SCHOOL, VADODARA

STD: XII **SAMPLE PAPER: 2** **MAX MARKS:100**
MATHEMATICS **TIME : 3 HRS**

General Instructions:

- All questions are compulsory
- The question paper consists of 29 questions divided into three sections A, B , C and D.
- Section A contains 4 questions of 1 mark each.
- Section B contains 8 questions of 2 marks each.
- Section C contains 11 questions of 4 marks each.
- Section D contains 6 questions of 6 marks each.

SECTION A

1. Find the number of binary operations on a set A where n(A) = 3 .
2. If $\tan^{-1} 2$, $\tan^{-1} 3$ are two angles of a triangle then find the third angle.
3. Construct a 3 x 4 matrix , whose elements a_{ij} are given by $a_{ij} = \frac{1}{2}\left|-3i + j\right|$.
4. If $\vec{a}$ is a unit vector perpendicular to $\vec{b}$ and $(\vec{a} + 3\vec{b}) \bullet (2\vec{a} - \vec{b}) = -10$,find $|\vec{b}|$

SECTION B

5. Define (i) injective function (ii) many –one function (iii) into function (iv) bijective function .
6. If A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$,prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$, $n \in N$.
7. If f(1) = 4 and f'(1) = 2, find the value of derivative of log $f(e^x)$ w.r.t. x at the point x=0 .
8. Find the integrating factor of the differential equation $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1} x}$.
9. Find the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$,where A (4 – 3 , 2) ,B (1, – 1 , – 1) , C (2,2,2) and D (3,3,3).
10. Find the distance of the plane through (1,1,1) and perpendicular to $\frac{x-1}{3} = \frac{y-1}{0} = \frac{z-1}{4}$ from the origin .
11. If A and B are two events such that P(A) = $\frac{1}{4}$; P(B) = $\frac{1}{2}$ and P(A∩B) = $\frac{1}{8}$, find P(not A and not B).
12. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1,2).

Section C

13. Without expanding evaluate $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix}$

OR

If $a+b+c \neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$,then prove a= b = c.

14. If $y = e^{ax} \cos bx$, prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.

15. Find the equation of the tangent to the curve $x^2 + 3y = 3$ which is parallel to $y - 4x + 5 = 0$

OR.

If $y = a \log x + bx^2 + x$ has its extreme values at x = -1 and x = 2, then find a and b.

16. Evaluate $\int_{\pi/6}^{\pi/3} \frac{1}{1+\tan x}\, dx$

17. Find the area of the region bounded by the curve $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

18. Solve the following differential equation : $\frac{dy}{dx} + y\cot x = x^2 \cot x + 2x$.

OR

Solve $(x^2 - y^2)\,dx + 2xy\,dy = 0$, given that y = 1 when x = 1.

19. If a unit vector $\vec{a}$ makes angle $\frac{\pi}{4}$ with $\hat{\imath}$, $\frac{\pi}{3}$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$,then find the component of $\vec{a}$ and the angle θ .

20. Show that the four points A(2,3,4) , B (- 3 , 5, 1) , C (4, -1 , 2) and D (2, 0,1) are coplanar .Find the equation of the plane containing them.

21. A box contains 13 bulbs out of which 5 bulbs are defective .3 bulbs are drawn one by one from the box with replacement .Find the probability distribution of the number of defective bulbs drawn.

22. Water is running out of a conical funnel at the rate of 5 cm^3/ sec.If the radius of the cone is 10 cm and altitude is 20 cm,find the rate at which the level of water dropping when it is 5 cm from the top.Explain the importance of water .What measures will you suggest to prevent wastage of water?

23. How many times must a man toss a fair coin so that the probability of having at least 1 head is more than 80% ?

Section D

24. Evaluate $\int_1^3 (3x+2)dx$ as limit of sum

OR

Evaluate $\int \frac{1}{x\left(6(\log x)^2 + 7\log x + 2\right)} dx$

25. (i) If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \frac{\pi}{2}$,prove that xy + yz + zx = 1.

(ii) If $\cos^{-1}(x/a) + \cos^{-1}(y/b) = \theta$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2} = \sin^2\theta$.

26. The sum of three numbers is 20.if we multiply the first number by 2 and add the second number to the result and subtract the third number we get 23 .By adding second and third numbers to three times the first number ,we get 46.Find the numbers by matrix method

OR

For the matrix A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Verify that $A^3 - 6A^2 + 9A - 4I = O$. hence find A^{-1}.

27. Show that a cone of greatest volume which can be inscribed in a given sphere is such that three times its altitude is twice the diameter of the sphere.Find the volume of the largest cone inscribed in the sphere of radius R.

OR

Show that the triangle of maximum area that can be inscribed in a given circle ia an equilateral triangle.

28. Find the angle between the line $\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-1}{-2}$ and the plane 3x + 4y + z + 5 =0 (ii) Find the shortest distance between the two parallel lines whose vector equations are $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and $\vec{r} = (\widehat{2\imath} + 4\hat{\jmath} + 5\hat{k}) + \mu(4\hat{\imath} + 6\hat{\jmath} + 4\hat{k})$.

29. David wants to invest Rs 12000 in bond A and B .According to the rules he has to invest at least Rs 2000 in bond A and at least Rs 4,000 in bond B .If the rate of interest on bond A is 8% per annum and rate of interest on bond B is 10 % per annum ,how much amount he should invest in each of the bonds A and B to earn maximum yearly income? Also ,determine the maximum yearly income.

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